Total No. of printed pages = 6

3 (Sem 2) MAT M2

2015

MATHEMATICS

(Major)

Theory Paper : 2.2

(Differential Equation)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1\times10=10$

   (a) Give a definition of an orthogonal trajectory of a family of curves.

   (b) What is an exact differential equation ?

   (c) When a first order differential equation is said to be homogeneous ?

   (d) Define a linear differential equation.

[Turn over

(e) Write down the complementary function of the differential equation $(D^2 + 1)y = e^{2x}$.

(f) Is the integrating factor of a differential equation $Mdx + Ndy = 0$ unique ?

(g) What is the geometrical interpretation of a differential equation ?

(h) The slope of a plane curve at a point is proportional to the product of the coordinates of that point. Express it by a differential equation.

(i) Write the form of a total differential equation.

(j) Write the particular integral of the differential equation $(D^2 - 3D + 2)y = e^x$.

2. Answer the following questions : $2\times5=10$

(a) Solve : $x^2(y-1)dx + y^2(x+1)dy = 0$

(b) Find the differential equation of

$y = e^x(a \cos x + b \sin x)$ where a and b are constants.

(c) Solve : $\dfrac{dy}{dx}+\dfrac{\sin 2y}{x}=x^3 \cos^2 y$

(d) Find the integrating factor of

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^2$$

(e) Construct the partial differential equation by eliminating a, b, c from $z = a(x+y) + b(x-y) + abt + c$.

3. Answer any *five* questions : 4×5=20

(a) Solve : $x dx + y dy + \left(1 + \frac{y^2}{x^2}\right)(y dx - x dy) = 0$.

(b) Find the orthogonal trajectories of the family of co-axial circles

$x^2 + y^2 + 2gx + c = 0$ where g is the parameter.

(c) Solve : $(1 + y^2)dx = (\tan^{-1} y - x)dy$

(d) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$$

(e) Solve : $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$, given that $x+\frac{1}{x}$ is one of its solutions.

(f) Solve : $\frac{dx}{y}=\frac{dy}{x}=\frac{dz}{xyz^2(x^2-y^2)}$

(g) Solve : $(D^3 + 1)y = \cos x$

4. Answer either (a) and (b) or (c) and (d) : 5+5=10

(a) Find the complete integral of $p^2x+q^2y = z$

(b) Solve : $y=px+\sqrt{a^2p^2+b^2}$

(c) Solve by Charpit's method

$z = px + qy + p^2 + q^2$

(d) Solve : $(D^2 - 2D + 1)y = x^2 e^{3x}$

5. Find the condition of exactness of a linear differential equation of order n. 6

Or

Find the necessary condition for integrability of the total differential equation $Pdx + Qdy + Rdz = 0$.

6. Answer either (a) and (b) or (c) and (d) : 6+6=12

(a) Find f(y) such that f(y)dx − zxdy − xylogy = 0 is integrable. Hence solve it.

(b) Eliminate the arbitrary function ϕ from the equation $\phi(x + y + z, x^2 + y^2 + z^2) = 0$.

What is the order of this differential equation ?

(c) Solve : $\dfrac{dx}{x(y^2 - z^2)} = \dfrac{dy}{y(z^2 - x^2)} = \dfrac{dz}{z(x^2 - y^2)}$

(d) Verify that the condition of integrability is satisfied by the equation $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$ and solve it.

7. Answer any *two* questions : 6×2=12

(a) Solve : $\dfrac{d^2y}{dx^2} - 2\tan x\dfrac{dy}{dx} + 3y = 2\sec x$ given that $y = \sin x$ is a part of its complementary function.

(b) Find the integral surface of the linear partial differential equation $(x-y)p + (y-x-z)q = z$ through the circle $z = 1$, $x^2 + y^2 = 1$.

(c) Reduce the differential equation $(px-y)(x-py) = 2p$ to Clairaut's form by the substitution $x^2 = u$ and $y^2 = v$ and find its complete primitive and its singular solution, if any.