2016

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatorics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) A bookshelf holds 6 different English books, 8 different French books and 10 different German books. In how many ways a book (of any one language) can be drawn from the bookshelf?

(b) Let X be a set and let A, B be any subsets of X. If $A \subset B$, then which of the following statements is false?

(i) $A \cap B' = \phi$

(ii) $A \cap B = A$

(iii) $A \cup B = B$

(iv) $A^c \subseteq B^c$

(c) There are 15 married couples in a party. Find the number of ways of choosing a woman and a man from the party such that the two are not married to each other.

(d) Can we draw a graph of 7 vertices such that the degree of each vertex is 3? If not, why?

(e) Draw the graph $\overline{K}_3 + \overline{K}_4$.

(f) Define cut point of a graph G.

(g) In a non-trivial tree with p vertices and q edges, p = ?

2. Answer the following questions : 2×4=8

(a) If X is any set and A is any subset of it, then show that

$$|X - A| = |X| - |A|$$

(b) Define enumerable set with suitable example.

(c) Can a graph containing a cycle of length 3 be a bipartite graph? Justify.

(d) Find the number of points and number of lines in—

(i) $K_5 + K_1$;

(ii) $K_{m,n}$.

3. Answer any *three* parts : 5×3=15

(a) Give a combinatorial proof of

$$C(n, r) = C(n-1, r) + C(n-1, r-1)$$ 5

(b) *(i)* Ten different paintings are to be allocated to n office rooms so that no room gets more than 1 painting. Find the number of ways of accomplishing this if $n = 14$.

(ii) There are n married couples at a party. Each people shakes hands with every person other than her or his spouse. Find the total number of handshakes. 2+3

(c) Define intersection graph. Show that every graph is an intersection graph. 1+4

(d) Define degree of a vertex of a graph. Let G be a (p, q) graph, all of whose vertices have degree K or $K+1$. If G has $p_K > 0$ vertices of degree K and p_{K+1} vertices of degree $K+1$, then prove that

$$p_K = (K+1)p - 2q$$ 1+4

(e) Show that every non-trivial tree has at least two end vertices (i.e., vertices with deg 1). 5

4. *(a)* For prescribed non-negative integers $\lambda_1, \lambda_2, \ldots, \lambda_m$, find the number of solutions in integers of the equation $x_1 + x_2 + \ldots + x_m = n$ with $x_i \geq \lambda_i$ for each i. 4

(b) Find the number of ways of choosing r positive integers from among the first n positive integers such that no 2 consecutive integers appear in the choice and the choice does not include 1 and n. 6

5. *(a)* *(i)* For any graph G, show that

$$\kappa(G) \leq \lambda(G) \leq \delta(G)$$

where the symbols have their usual meaning. 7

(ii) Draw a graph for which $\kappa = 2$, $\lambda = 3$ and $\delta = 4$. 3

Or

(b) *(i)* For all integers a, b, c such that $0 < a \leq b \leq c$, show that there exists a graph with $\kappa(G) = a$, $\lambda(G) = b$ and $\delta(G) = c$. 6

(ii) Define connectivity function of a graph G. Show that this function is strictly decreasing function. 4

6. Prove the equivalence of the following statements : 10

(a) G is Eulerian

(b) Every vertex of G has even degree

(c) The set of edges of G can be partitioned into cycles

Or

(a) If for all vertices v of a graph $G(p, q)$, $\deg v \geq p/2$, where $p \geq 3$, then show that G is Hamiltonian. 6

(b) Give an example of a graph which is both Eulerian and Hamiltonian. 2

(c) How many Hamiltonian cycles are there in K_4? 2

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