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MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following questions :   1×10=10

   (a) Write the nth derivative of $\sin^3 x$.

   (b) If $f(x, y) = 3x^2y + 2xy^2$, find $f_x(1, 2)$.

   (c) State Euler's theorem on homogeneous function of degree n for two variables.

   (d) Write the subtangent of the curve $y^2 = 4ax$.

   (e) Define asymptotes.

   (f) Write the value of $\int_{-a}^{a} x^3 \sqrt{a^2 - x^2}\, dx$.

   (g) Define point of inflexion.

   (h) For a pedal curve $p = r \sin \phi$, write the formula for radius of curvature.

(Turn Over)

(i)   Write down the reduction formula for

$$\int \tan^n x\, dx$$

(j)   What is a cusp?

2. Answer the following questions :   2×5=10

(a)   Find nth derivative of $\dfrac{1}{a^2 - x^2}$:

(b)   If $u = x^2 \tan^{-1} \dfrac{y}{x} - y^2 \tan^{-1} \dfrac{x}{y}$, find

$$\frac{\partial^2 u}{\partial x \partial y}$$

(c)   The tangent of the curve $y^2 = 4a\left\{x + \sin \dfrac{x}{a}\right\}$

at $(x_1,\ y_1)$ is parallel to x-axis. Show that
$$\cos(x_1 / a) = -1$$

(d)   Evaluate $\displaystyle\int_0^\pi x \sin x \cos^2 x\, dx$.

(e)   Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

3. Answer the following questions :

(a)   (i)   If $u = e^{xyz}$, show that
$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz} \qquad 3$$

(ii) Find the pedal equation of the curve
$$x^2 + y^2 = 2ax \qquad 2$$

(b)   Derive a reduction formula for $\int \cos^n x\, dx$.   5

(Continued)

4. Answer either *(a)* or *(b)* :

   (a) (i) Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on

   $$x^2 y^2 = x^2 - y^2 \qquad 5$$

   (ii) Evaluate $\displaystyle\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}}.$ 5

   (b) (i) Evaluate $\displaystyle\int \frac{dx}{3+5\cos x}.$ 5

   (ii) Evaluate $\displaystyle\int \sqrt{\frac{x-3}{x-4}}\, dx.$ 5

5. Answer the following questions :

   (a) If $y = [x + \sqrt{1+x^2}]^m$, find the nth derivative of y for $x = 0$.   5

   (b) Find the perimeter of the circle

   $$x^2 + y^2 = a^2 \qquad 5$$

6. Answer either *(a)* or *(b)* :

   (a) (i) If $u = x\,\phi(y/x) + \psi(y/x)$, prove that

   $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 y}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0 \qquad 5$$

   (Turn Over)

(ii) Find the volume of the solid generated by the revolution of the curve $(a-x)y^2 = a^2x$ about its asymptote. 5

(b) *(i)* Find the asymptotes of the curve
$$x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0 \qquad 5$$

(ii) Trace the curve $y = x^3$. 5

7. Answer the following questions :

(a) Show that points of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line. 5

8. Answer either *(a)* or *(b)* :

(a) Derive a reduction formula for
$$\int \sin^m x \sin nx \, dx$$

Hence evaluate
$$\int_0^\pi \sin^m x \sin nx \, dx \qquad 7+3=10$$

(b) What are the double points? Examine the nature of double points of the curve
$$2(x^3 + y^3) - 3(3x^2 + y^2) + 12x = 4 \qquad 2+8=10$$

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