3 (Sem–1/CBCS) MAT HC 1

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT–HC–1016

(Calculus)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times7=7$

(a) Write down the nth derivative of $y = \log x$.

(b) The point $P(c, f(c))$ on the graph of $f(x)$ is such that $f''(c)=0$. Does it necessarily imply that P is an inflection point on the graph ?

Contd.

(c) Write down the value of $\underset{x \to +\infty}{lt} x \sin \frac{1}{x}$.

(d) Find the domain of the vector function

$$\vec{F}(t) = (1-t)\hat{i} + \sqrt{-t}\,\hat{j} + \frac{1}{t-2}\hat{k}$$

(e) Write one basic difference between the disk/washer and shell method for computing volume of revolution.

(f) What is the direction of velocity of a moving object on its trajectory.

(g) The velocity of a particle moving in space is $\vec{V}(t) = e^t \hat{i} + t^2 \hat{j}$. Find the direction of motion at time $t = 2$.

2. Answer the following questions : $2\times4=8$

(a) Applying L.Hopital's rule, evaluate

$$\underset{x \to \pi/4}{lt} (1 - \tan x).\sec 2x$$

(b) Write down the parametric equation of a line that contains the point (3,1,4) and is parallel to the vector $\vec{v} = -\hat{i} + \hat{j} - 2\hat{k}$.

(c) Find the area of the surface generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(d) Explain briefly why the acceleration of an object moving with constant speed is always orthogonal to the direction of motion.

3. Answer ***any three*** of the following questions : 5×3=15

(a) If $y = \cos(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} + (m^2 - n^2)y_n = 0.$$

Hence find $y_n(0)$. 3+2=5

(b) Sketch the graph of a function f with all the following properties : 5

(i) the graph has $y=1$ and $x=3$ as asymptotes

(ii) f is increasing for $x<3$ and $3<x<5$ and decreasing elsewhere

(iii) the graph is concave up for $x<3$ and concave down for $3<x<7$

(iv) $f(0)=4=f(5)$ and $f(7)=2$

(c) Sketch the graph of $y = \frac{3x-5}{x-2}$ identifying the locations of intercepts, concavity and inflection points (if any) and asymptotes. 5

(d) Obtain the reduction formula for $\int tan^n x \, dx$.

Hence evaluate $\int_0^{\pi/4} tan^5 x \, dx$ 3+2=5

(e) The position vector of a moving object at any time t is given by $\vec{R}(t) = t\hat{i} + e^{t}\hat{j}$.

Find the tangential and normal components of the object's acceleration. 5

4. Answer ***any three*** of the following questions : 10×3=30

(a) A firm determines that x units of its product can be sold daily at rupees p per unit where $x = 1000 - p$. The cost of producing x units per day is $C(x) = 3000 + 20x$. Then —

(i) Find the revenue function $R(x)$. 2

(ii) Find the profit function $p(x)$. 2

(iii) Assuming that production capacity is atmost 500 units per day, determine how many units the company must produce and sell each day to maximize profit. 3

(iv) Find the maximum profit. 2

(v) What price per unit must be charged to obtain maximum profit ? 1

(b) When is an object said to move in central force field ? Derive Kepler's 2nd law of motion, assuming that planetary motion occurs in central force field. 2+8=10

(c) (i) Find the length of the arc of the astroid $x^{2/3}+y^{2/3}=1$ lying in the positive quadrant. 3

(ii) Using cylindrical shell method, find the volume of the solid formed by revolving the region bounded by the parabola $y=1-x^2$, the y- axis, and the positive x-axis, about y-axis. 4

(iii) Find the surface area generated when the polar curve

$$r = 5,\ 0 \le \theta \le \pi/3$$

is revolved about x-axis. 3

(d) (i) Find the volume generated by disk/washer method, when the region bounded by $y = x$, $y = 2x$ and $y = 1$ is revolved about the x-axis 5

(ii) A particle moves along the polar path (r, θ) where

$$r(t) = 3 + 2\sin t,\ \theta(t) = t^3.$$

Find the velocity $\vec{v}(t)$ and acceleration $\vec{A}(t)$ in terms $\hat{u}_r$ and $\hat{u}_\theta$. 5

(e) (i) Evaluate $\lim\limits_{x \to 0} (1 + \sin x)^{1/x}$. 3

(ii) Examine the existence of vertical tangent and cusp of the graph of $y = (x - 4)^{2/3}$. 3

(iii) A projectile is fired from ground level at an angle of 30° with muzzle speed 110 *ft/sec*. Find the time of flight and the range. 4

(f) (i) Obtain the reduction formula for

$$\int \cos^n x \, dx.$$

Hence evaluate $\int \cos^5 x \, dx$.

3+2=5

(ii) Find the unit tangent vector $\vec{T}(t)$ and principal unit normal vector $\vec{N}(t)$ at each point on the graph of vector function

$$\vec{R}(t) = (3\sin t, 4t, 3\cos t)$$ 5
