3 (Sem–3/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer **any seven** of the following questions : 1×7=7

    (a) Define the singular point of a second order linear differential equation.

    (b) If $P_n(x)$ and $Q_n(x)$ are two independent solutions of Legendre equation, then write the general solution of the Legendre equation.

    (c) Give *one* example where Hermite polynomial is used in physics.
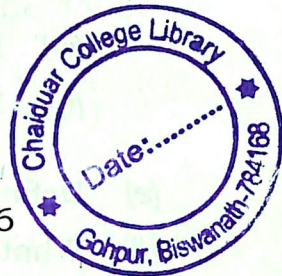
(d) The function $P_n(1)$ is given as

  (i)   zero

  (ii) -1

  (iii) $P_n(-1)$

  (iv) 1

          (Choose the correct option)

(e) Define trace of a matrix.

(f) What is the rank of a zero matrix ?

(g) Define self-adjoint matrix.

(h) What do you mean by eigenvector ?

(i) Which one of the following represents an equation of a vibrating string ?

  (i) $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$

  (ii) $\dfrac{\partial y}{\partial t} = c \dfrac{\partial y}{\partial x}$

  (iii) None of the above
          (Choose the correct option)

(j) Write the Laplace equation spherical polar co-ordinate system.

(k) Define gamma function.

(l) State the Dirichlet condition for Fourier series.

2. Answer **any four** of the following questions :
                                    $2\times4=8$

(a) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(b) If $\displaystyle\int_{-1}^{+1} P_n(x)dx = 2$, find the value of n.

(c) If A and B are Hermitian matrices, show that $AB + BA$ is Hermitian whereas $AB - BA$ is skew-Hermitian.

(d) Verify that $(AB)^T = B^T A^T$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

(e) Given matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \ \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

  show that $\sigma_1\sigma_2 - \sigma_2\sigma_1 = 2i\sigma_3$.

(f) Using the property of gamma function evaluate the integral

$$\int_0^\infty x^4 e^{-x} dx$$

(g) Write the degree and order of the following partial differential equations :

(i) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

(ii) $\left(\dfrac{\partial u}{\partial x}\right)^3 + \dfrac{\partial u}{\partial t} = 0$

(h) Find the value of a_0 of the Fourier series for the function $f(x) = x\cos x$ in the interval $-\pi < x < \pi$.

3. Answer **any three** of the following questions :

$$5 \times 3 = 15$$

(a) (i) Why is the function

$\left(1 - 2xh + h^2\right)^{-\frac{1}{2}}$ known as a generating function of Legendre polynomial ?

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(ii) Show that

$$\left(1 - 2xh + h^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x)h^n$$

where $P_n(x)$ is the Legendre polynomial.

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(b) Evaluate explicitly the Legendre's polynomials $P_2(x)$ and $P_3(x)$.

$2\frac{1}{2} + 2\frac{1}{2} = 5$

(c) Write the recursion formula for gamma function. Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = 1.772$$

(d) What is diagonalize matrix ? Diagonalize the following matrix :

$$1 + 4 = 5$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(e) Express the matrix :

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$ as a sum of symmetric and skew-symmetric matrix.

(f) What is adjoint of a matrix ? For the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ verify the theorem

$$A \cdot (Adj A) = (Adj A) \cdot A = |A| \cdot I$$

where I is unit matrix.

$$1 + 4 = 5$$

Contd.

(g) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r x^{k+r}, \text{ show that the allowed}$$

values of k are zero and one only.

(h) Find the Fourier series representing

$$f(x) = x, \quad 0 < x < 2\pi$$

4. Answer **any three** of the following questions : $10 \times 3 = 30$

(a) (i) Verify that the matrix

$$A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \text{ is orthogonal.} \quad 2$$

(ii) Verify Cayley-Hamilton theorem for

the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and also

find A^{-1}. $5+3=8$

(b) Obtain the power series solution of the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

(c) (i) Obtain the following orthogonality property of Legendre polynomial :

$$\int_{-1}^{+1} P_n(x)P_m(x)dx = 0 \text{ for } m \neq n \qquad 6$$

(ii) Show that

$$H_0(x) = 1 \text{ and } H_1(x) = 2x \qquad 2+2=4$$

(d) Prove the following recurrence relations : $4+3+3=10$

(i) $n P_n = (2n-1)x P_{n-1} - (n-1)P_{n-2}$

(ii) $x P_n' - P_{n-1}' = n P_n$

(iii) $2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x)$

(e) What is periodic function ? Express the periodic functions in a series of sine and cosine functions. What are Fourier coefficients ? Determine the Fourier coefficients. $1+1+1+7=10$

(f) (i) Using the method of separation of variables, solve : 6

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ where } u(x,0) = 6e^{-3x}$$

Contd.

(ii) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 4

(g) *(i)* If $H_n(x)$ be the polynomial of Hermite differential equation, prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x)\, dx = 2^n \sqrt{\pi}.\, n!$$ 7

(ii) Prove that the following matrix is unitary :

$$\begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$$ 3

(h) Deduce the one dimensional wave equation of transversely vibrating string under tension T. Solve the equation by the method of separation of variables. 7+3=10