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3 (Sem–5/CBCS) PHY HE 3

2022

PHYSICS

(Honours Elective)

Paper : PHY–HE–5036

(*Advanced Mathematical Physics–I*)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : $1\times7=7$

   (a) What are the basis and dimension of a vector space ?

   (b) Define subspace. Give *one* example.

   (c) Give the definition of a homomorphic group.

   (d) Define Hooke's law of elasticity in tensorial notation.

Contd.

(e) Write the order of tensor C if $C = a_{pr}a_{rst}$.

(f) Write the relation between Alternate tensor and Kronecker tensor.

(g) State the Quotient law of tensors.

(h) Give *two* examples of zero order tensor.

(i) Define linear dependence and linear independence of a finite set of vectors.

(j) Write the transformation law of the tensor A^{p}_{qr}.

(k) Define Minkowski space.

(l) What is binary relation?

2. Answer ***any four*** of the following questions: 2×4=8

(a) Show that gradient of a scalar field is a covariant tensor of rank 1.

(b) Write scalar triple product $\vec{A}\cdot(\vec{B}\times\vec{C})$ using suffix notation.

(c) Find the second order antisymmetric tensor associated with the vector $2\hat{i}-3\hat{j}+\hat{k}$.

(d) Show that Kronecker delta is an isotropic mixed tensor of order 2.

(e) Determine whether or not the vector $W = [1, 7, -4]$ belongs to the subspace of R^3 spanned by $W_1 = [2, -1, 1]$ and $W_2 = [1, -3, 2]$.

(f) Prove that eigenvalue of a matrix A is same as that of the transpose matrix A^T.

(g) Find the bases and the dimension of the subspaces of S of R^3 defined by $S = \{[a, b, 0] \mid a, b \in R\}$.

(h) Using tensor notation, show that

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$

3. Answer **any three** of the following questions : 5×3=15

(a) Give the definition of a group. What do you mean by sub group and invariant sub group? 3+2=5

(b) Given $\{W_1, W_2, W_3\}$ is a linearly independent set of vectors. Show that $\{(W_1 + W_2), (W_3 + W_2), (W_3 + W_1)\}$ is also linearly independent.

(c) If A_p and B^p are the components of a co-variant and contravariant vector respectively, then prove that the sum $A_p B^p$ is invariant.

(d) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}$. Also find A^{-1}.

3+2=5

(e) Determine the identity element and inverse for the binary operation

$$(a, b) * (c, d) = (ac, bc + d)$$

(f) What is alternating tensor? Prove that

$\varepsilon_{iks}\varepsilon_{mps} = \delta_{im}\delta_{kp} - \delta_{ip}\delta_{km} = 0$ 2+3=5

(g) "The inner product of tensors can be thought of as outer product followed by contraction." Illustrate with example.

(h) Diagonalize the matrix

$$P = \begin{bmatrix} 1 & 1+i \\ 1-i & 0 \end{bmatrix}$$

4. Answer ***any three*** of the following questions : 10×3=30

(a) Using tensor, prove the following vector identities 3+3+4=10

(i) $(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{A}(\vec{B} \cdot \vec{C})$

(ii) $\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = (\vec{\nabla} \times \vec{A}) \cdot \vec{B} - (\vec{\nabla} \cdot \vec{B}) \cdot \vec{A}$

(iii) $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 A$

(b) (i) Find the eigenvalues and eigenvectors of the matrix 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

(ii) Show that any tensor A_{pq} can be expressed as a sum of two tensor, one is symmetric and the another is skew-symmetric. 4

(c) (i) Solve the coupled differential equations : 6

$y' = y + z$ and $z' = 4y + z$;

where $y(0) = z(0) = 1$

(ii) Show that

$\varepsilon'_{hsu} = \varepsilon_{hsu}$, i.e., ε_{hsu} is an isotropic tensor and

$\varepsilon_{hku}\varepsilon_{pcm}\delta_{kc}\delta_{um} = 2\delta_{hp}$ 2+2=4

(d) (i) What is inertia tensor ? Show that the inertia tensor is a symmetric tensor of order 2. 2+4=6

(ii) If A and B are Hermitian matrices show that $(AB + BA)$ is Hermitian and $(AB - BA)$ is skew-Hermitian. 4

(e) (i) What is metric tensor ? Calculate the co-efficients of metric tensor in 3D Euclidean space for Cartesian, cylindrical and spherical polar co-ordinate. 2+2+2+2=8

(ii) If

$$(ds)^2 = 3(dx^1)^2 + 5(dx^2)^2 - 4(dx^1)(dx^2)$$

find g_{qr}. 2

(f) (i) Find whether the set of vectors $[\alpha, \beta, \gamma]$ in R^3, such that $\alpha + \beta + \gamma = 0$ forms a subspace of R^3. 5

(ii) Show that the modulus of each eigenvalue of a unitary matrix is unity. 5

(g) (i) Show that

$$\bar{\nabla} \cdot \vec{A} = A^i_{;i} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \left(\sqrt{g} A^i \right)$$ 8

(ii) Write $\nabla^2 \phi$ in tensor notation. 2

(h) (i) What is abelian group ? Prove that the set I of all integers with the binary operation $*$ defined by $x * y = x + y + 1$ forms a group. 2+5=7

(ii) If $A^\lambda B_{\mu\nu}$ is a tensor for all contravariant tensors A^λ then show that $B_{\mu\nu}$ is also a tensor. 3

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